BRIEF REPORTS

Brief Reports are accounts of completed research which do not warrant regular articles or the priority handling given to Rapid Communications; however, the same standards of scientific quality apply. (Addenda are included in Brief Reports.) A Brief Report may be no longer than four printed pages and must be accompanied by an abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.

Langmuir waves in a dusty plasma with variable grain charge

Jitesh R. Bhatt*
Theoretical Physics Division, Physical Research Laboratory, Navarangpura, Ahmedabad 380 009, India
(Received 11 June 1996)

Using the equations of self-consistent charge dynamics for the grain charge fluctuations, electron plasma oscillations in a dusty plasma are studied. It is shown that the instability in longitudinal plasma oscillations disappears if the self-consistent grain charge dyanmics is taken into account. The relation of this result with previous work is discussed. [S1063-651X(97)08901-0]

PACS number(s): 52.25.Mq, 52.25.Fi, 52.35.Fp

The physics of dusty plasma has recently been under extensive investigations due to applications in space plasmas, interstellar environment, laboratory plasma devices, etc. (for a recent review see Ref. [1]). In many diverse physical systems, one may often encounter a situation where highly charged massive grains of submicrometer to micrometer size, immersed in a plasma environment. A grain in a plasma can acquire a net electric charge due to various physical processes such as its collisions with the other plasma particles and photoionization. Collective effects due to the dust grains become important if the intergrain spacing is much less than the plasma Debye length. Such a plasma is called a dusty plasma. The collective behavior of dusty plasmas has been investigated by many workers (e.g., see Refs. [2–6]) by treating the mass and the charge of the dust grains as constant and uniform. It is found that the presence of massive and highly charged low-density grains can introduce new length and time scales in the plasma. It should be stressed that such studies are qualitatively very similar to those of negative-ion plasmas.

However, in a realistic situation, the charge on the dust grains may not be constant, but varies with wave motion. Since the current falling on the dust grain surface fluctuates due to the collective wave motion, the grain charge also fluctuates with it. This feature of the dusty plasma can make it qualitatively different from a negative-ion plasma. However, the studies investigating the effects of dust charge variations on the plasmas' collective behaviors have only recently begun [7–15]. It is found that the charge-varying dust grains can provide a dissipative background in the plasma. This may cause the damping of the normal modes in dusty plasmas and in some circumstances introduces instability related to the dissipative background.

The effect of the grain charge variations on the acoustic branch of the dusty plasma dispersion relation is reasonably well understood and rests on a firm footing. However, its effect on the high-frequency branch, i.e., on the longitudinal waves, is not well understood yet. There remains uncertainty about the nature of Langmuir waves in the presence of the grain charge fluctuations. In particular, various studies, within the framework of both fluid theory and kinetic theory, predict the instability of longitudinal plasma oscillations related with the grain charge fluctuations [13,14]. But the origin and the nature of the instability remain unclear. In such studies, the dust charge dynamics have been treated in an *ad hoc* fashion and in the absence of the quasineutrality it may be inconsistent with the charge conservation [15]. A proper formulation of the self-consistent charge dynamics was recently done by Bhatt and Pandey [15] within the fluid theory framework. In such a formulation, sink terms are required in the electron and ion number continuity equations and the appropriate friction terms in the equation of motion to comply with the conservation laws. For the instances when the charge dynamics is treated in an *ad hoc* fashion no such additional terms are included in the dynamical equations. In the present paper, I apply the self-consistent charge dynamics to study the longitudinal plasma oscillations. It will be shown in what follows that longitudinal plasma oscillations are damped when the full self-consistent charge dynamics is taken into account. In the absence of any additional term in the dynamical equations, all the previous results indicating the instability of Langmuir waves can be recovered. Finally, the reasons for such instability will be discussed.

Basic linearized equations for the number conservation of the electron (e), ion (i), and dust (d) fluids are

$$\partial_t n_{e1} + \vec{\nabla}\cdot(n_{e1}\vec{V}_{e0} + n_{e0}\vec{V}_{e1}) = -\beta_{e0} n_{e1} - \beta_{e1} n_{e0}, \quad (1)$$

$$\partial_t n_{i1} + \vec{\nabla}\cdot(n_{i1}\vec{V}_{i0} + n_{i0}\vec{V}_{i1}) = -\beta_{i0} n_{i1} - \beta_{i1} n_{i0}, \quad (2)$$

$$\partial_t n_{d1} + \vec{\nabla}\cdot(n_{d1}\vec{V}_{d0} + n_{d0}\vec{V}_{d1}) = 0, \quad (3)$$

*Electronic address: jeet@prl.ernet.in

where the subscripts 0 and 1 are used to denote equilibrium and perturbed quantities, respectively. $n_\alpha, \vec{V}_\alpha$ ($\alpha = e, i$, or d) are, respectively, the number density and velocity of αth species, whereas β_e and β_i denote attachment frequencies of the electrons and ions to the dust. The right-hand sides of Eqs. (1) and (2) describe the loss in the electron and ion densities. The attachment frequency of the the electrons (ions) may vary due to the change in the electron (ion) current falling on the dust grains and also due to the change in the electron (ion) and the dust grain densities. This results in the presence of two terms on the right-hand sides of Eqs. (1) and (2). The dust charge variation equation is then obtained from the charge conservation equation

$$\partial_t Q_{d1} + \vec{V}_{d0} \cdot \vec{\nabla} Q_{d1} = -\frac{e}{n_{d0}}[(\beta_{e0} n_{e1} + \beta_{e1} n_{e0}) - (\beta_{i0} n_{i1} + \beta_{i1} n_{i0})]. \quad (4)$$

The equations of motion for the electron, ion, and dust fluids are

$$m_a n_a \frac{d\vec{V}_a}{dt} = -\vec{\nabla} P_a + q_a n_a \vec{E} - m_a n_a \beta_a (\vec{V}_a - \vec{V}_d), \quad (5)$$

$$m_d n_d \frac{d\vec{V}_d}{dt} = Q_d n_d \vec{E} + \Sigma_a m_a n_a \beta_a (\vec{V}_a - \vec{V}_d), \quad (6)$$

where m_a ($a = e, i$) is the mass of the species a. Poisson's equation is

$$\vec{\nabla} \cdot \vec{E} = 4\pi e (n_i - n_e) + 4\pi Q_d n_d. \quad (7)$$

Equations (1)–(7) can be closed by choosing the appropriate equation of state and relating the electron (ion) attachment frequencies β_{a1}, $a = e$ or i, to the other known physical quantities. The perturbations in the attachment frequencies may be computed by using the kinetic theory arguments. The ''average'' perturbations in the attachment frequencies can be defined as

$$\beta_{a1} = \frac{\int \nu_{a1}(v) f_{a0} dv}{\int f_{a0} dv}, \quad (8)$$

where f_{a0} is the equilibrium distribution function of the electron ($a = e$) or the ion ($a = i$) and $\nu_{a1}(v)$ is the microscopic attachment frequency of the electron or ion with the dust, from Vladimirov [12], and is given by

$$\nu_{a1}(v) = \int \sigma_a v f_{d1} dq. \quad (9)$$

Here σ_a is the electron (ion) charging cross section with the dust and is defined as

$$\sigma_a = \pi a^2 \left(1 - \frac{2 e_a q}{a m_a v^2} \right) \quad (10)$$

if $2e_a q / a m_a v^2 < 1$ and $\sigma_a = 0$ if $2 e_a q / a m_a v^2 \geqslant 1$. m_a is the electron or ion mass. a is the radius of the grains and q is the charge of the dust particles. f_{d1} is the perturbed distribution function of the grains. For the high-frequency response of a dusty plasma, the spatial or velocity distribution of the dust grains may not change very much over the time scales of interest. The perturbations in the distribution function still may arise due to the charge variation and can be written as

$$f_{d1} = n_{d0}[\delta(q - Q_{d0} - Q_{d1}) - \delta(q - Q_{d0})], \quad (11)$$

where $Q_{d0} = -eZ_d$ is the equilibrium value of the grain charge. Q_{d1} is the macroscopic fluctuation in the grain charge value and $Q_{d1} \ll |Q_{d0}|$. Using Eqs. (10) and (11), $\nu_{a1}(v)$ can readily be evaluated as

$$\nu_{a1} = \left(\frac{\pi a e n_{d0}}{m_a} \right) \frac{Q_{d1}}{v}. \quad (12)$$

Further, taking f_{a0} as Maxwellian, i.e.,

$$f_{a0} = \left(\frac{m_a}{2\pi k T_a} \right)^{3/2} \exp\left[-\frac{2 m_a v^2}{k T_a} \right],$$

and using Eq. (12), one can get from Eq. (8)

$$\beta_{a1} = 2\sqrt{\left(\frac{2\pi}{m_a k T_a} \right)} a e n_{d0} Q_{d1}. \quad (13)$$

This relation, together with the equation of state, closes the system of equations (1)–(7). It can be seen from Eq. (13) that when electron and ion temperatures are comparable $\beta_{i1} \ll \beta_{e1}$. As already mentioned, the aim of this paper is to study the longitudinal plasma oscillations; the dust and ion are regarded as stationary fluids with $V_{i1}, V_{d1} = 0$ and $n_{d1} = 0$. The perturbations in the ion density can be neglected from the charge dynamics equation if $\beta_{i1} \ll \beta_{e1}$ and thus the ion current may not contribute in the charging process over the time scale of longitudinal plasma oscillations. Thus Eqs. (2) and (3) are dropped. The charge dynamics equation and the equation of motion of the electrons can be written as

$$\partial_t Q_{d1} = -\frac{e}{n_{d0}}(\beta_{e0} n_{e1} + \beta_{e1} n_{e0}), \quad (14)$$

$$m_e n_{e0} \frac{d\vec{V}_{e1}}{dt} = -e n_{e0} \vec{E}_1 - \vec{\nabla} P_{e1} - m_e n_{e0} \beta_{e0} \vec{V}_{e1} - m_e n_{e1} \beta_{e1} \vec{V}_{e0} - m_e n_{e1} \beta_{e0} \vec{V}_{e0} \quad (15)$$

and Poisson's equation can be written as

$$\vec{\nabla} \cdot \vec{E}_1 = -4\pi e n_{e1} + 4\pi Q_{d1} n_{d0}. \quad (16)$$

Equations (1) and (13)–(16), together with the equation of state $P_{e1} = n_{e1} T_e$, form a closed set and will be used to study the longitudinal plasma oscillations. Considering all the perturbed quantities to vary like $\exp[-i(\omega t - \vec{k} \cdot \vec{x})]$, the follwing dispersion relation can be obtained:

$$\left[(\omega + i\beta_{e0})^2 - k^2 V_{te}^2 - \omega_{pe}^2 \left(1 + i\frac{\beta_{e0}}{\omega} \right) - i\beta_{e0} \vec{k} \cdot \vec{V}_{e0} \right] = G(\omega) \left[\frac{\omega_{pe}^2}{\omega} - \vec{k} \cdot \vec{V}_{e0} - \overline{\omega -} \, i\beta_{e0} \right], \quad (17)$$

where, $\overline{\omega}=\omega-i\beta_{e0}$. $\omega_{pe}^2=4\pi e^2 n_{e0}/m_e$ and $V_{te}^2=T_e/m_e$ are the electron plasma frequency and the thermal velocity, respectively. $G(\omega)$ is given by

$$G(\omega)=\frac{\frac{1}{\sqrt{2\pi}}\left(\frac{a}{\lambda_{\mathrm{De}}}\right)\frac{\omega_{pe}}{\omega}}{1+i\frac{1}{\sqrt{2\pi}}\left(\frac{a}{\lambda_{\mathrm{De}}}\right)\frac{\omega_{pe}}{\omega}}\beta_{e0}. \qquad (18)$$

First, consider the case in which there is no streaming, i.e., $V_{e0}=0$,

$$(\omega+i\beta_{e0})^2=(k^2V_{te}^2+\omega_{pe}^2)+i\frac{\omega_{pe}^2\beta_{e0}}{\omega+i\alpha}, \qquad (19)$$

where $\alpha=(1/\sqrt{2\pi})(a/\lambda_{\mathrm{De}})\omega_{pe}$ and λ_{De} is the electron Debye length. Equation (19) is obtained by considering $\alpha,\beta_{e0}\ll\omega$ and retaining terms only up to first order in α and β_{e0} on the right-hand side of the equation. Before analyzing Eq. (19) further, we first estimate the values of the parameters α and β_{e0}. The magnitude of the equilibrium grain charge $Q_{d0}=-eZ_d$ can be found from the wll-known relation [16]

$$\exp(-z)=\left(\frac{1+P}{\sqrt{\tau}}\right)\sqrt{\frac{m_e}{m_i}}(\tau+z).$$

Here $z=e^2Z_d/aT_e$ and $\tau=T_e/T_i$. The parameter P signifies the ratio between the dust space-charge density and the electron space-charge density. P can be estimated from the relation of equibrium number densities, i.e., $n_{i0}=n_{e0}+Z_dnd0=n_{e0}(1+P)$. The attachment frequency β_{e0}, using Ref. [12], can be found to be $\beta_{e0}=(1/\sqrt{2\pi})(\omega_{pi}^2a/V_{Ti})P(\tau+z)$. Here ω_{pi} and V_{Ti} represent the ion plasma frequency and the thermal velocity respectively. As an example, similar to Ref. [13], we consider the case of a hydrogen plasma with $T_e=10T_i=1$ eV, $n_{e0}=0.9n_{i0}=10^{12}$ cm^{-3}, and $a=1$ μM. For this case z can be estimated to be 1.8. From this one can estimate $\alpha/\omega_{pe}\sim 10^{-1}$ and $\beta_{e0}/\omega_{pe}\sim 10^{-1}$. This is consistent with the assumption $\alpha,\beta_{e0}\ll\omega\sim\omega_{pe}$.

The terms with β_{e0} on the left-hand side of Eq. (19) arise due to their presence in the number continuity equation and in the equation of motion of the electrons. In the absence of β_{e0} terms on the left-hand side of Eq. (19), it is similar to the dispersion relation of longitudinal oscillations obtained in Ref. [14] and describe unstable mode. Equation (19) is a cubic equation in ω and can be solved analytically; however, the form of the solution is very complicated. It is instructive to solve it perturbatively by writing $\omega\simeq\omega_0+\omega_1$ with $\omega_0\gg\omega_1$ and retaining the terms up to first order in ω_1 and β_{e0}. Thus, from Eq. (19) one may obtain

$$\omega_0=\pm\sqrt{(k^2V_{te}^2+\omega_{pe}^2)}, \qquad (20)$$

$$\omega_1=-i\beta_{e0}+i\frac{\beta_{e0}}{2}\frac{\omega_{pe}^2}{k^2V_{te}^2+\omega_{pe}^2}. \qquad (21)$$

The first term on the right-hand side of Eq. (21) describes the damping of plasma oscillations and arises due to the presence of β_{e0} terms in Eqs. (1) and (15). For instance, when there is no term with an attachment frequency in Eqs. (1) and (15), the first term on the right-hand side of Eq. (21) is absent. In this case, Eq. (21) describes the instability of the plasma oscillations and the growth rate that one obtains is exactly the same as that in Ref. [13] using the kinetic theory approach. However, it should be emphasized that, using the equations of self-consistent charge dynamics, one cannot consistently neglect the $-i\beta_{e0}$ term from Eq. (21). Since $\omega_{pe}^2/(k^2V_{te}^2+\omega_{pe}^2)\leqslant 1$, the second term on the right-hand side of Eq. (21) is always less than the first term and therefore the longitudinal waves are damped. It ought to be mentioned that in Ref. [9] longitudinal plasma oscillations were studied by treating the grain charge dynamics in an *ad hoc* fashion and the damping of plasma oscillations was found. But in this work it is assumed that the grain charge variation is essentially driven by a thermally equilibrated electron number density. However, in the presence of an electron plasma oscillation such an assumption may not be justified.

Thus all the models of a charge varying dusty plasma without having charging frequency terms in the electron dynamical equation predict the instability in longitudinal plasma oscillations. The reason for this instability may be the following: The total number of electrons is conserved as the number continuity equation does not contain any term with the charging frequency. Thus the electric-field fluctuations produced by the grain charge variation is not due to the number of electrons lost or gained from it. Since electron density fluctuatons drive grain charge fluctuations, the phase of the electric field generated by the grain charge variation is the same as that due to electron density fluctuations. Such an electric field can accelerate the electron and provide a source of free energy. The net charge in such a system is not conserved. However, when one considers the low-frequency response of a dusty plasma, the quasineutrality condition does not allow such an electric field to be created. Therefore, in the low-frequency regime, where the quasineutrality condition is respected, the models that treat the dust charge dynamics in an *ad hoc* fashion do not give any such instability [8–10]. However, the inclusion of the self-consistent charge dynamics can strongly modify the instability and the wave spectra [7,11,15].

But, in a realistic situation, as the electrons take part in the charging process, their number is not conserved. Moreover, they lose their momentum in charging the dust grains. In this process the electrons lose more energy than the energy they build up in the form of the dust electric field. This electric field then does not provide a source of the free energy. These features of the charging process are included in the self-consistent grain charge dynamics [15]. Therefore, the models that contain the charging frequency terms in the electron dynamics, including the present one, do not give any such instability [11,12]. However, if the electrons taking part in the charging process are externally compensated so that their number is conserved and their momentum is unaffected then such an instability might occur. But, this will violate the charge conservation and the reason for the instability is the same as above.

Next, the charging frequency in a dusty plasma can be higher than the collision frequencies by the other collision process [11]. Therefore, the damping caused by the charging process gives the most dominant contribution as compared to other collisional processes. However, the Landau damping can also be operative on longitudinal plasma oscillations and the damping frequency for it is given by

$$\Gamma_L = -\omega_{pe}\frac{\sqrt{\pi}}{\sqrt{2}k^3\lambda_e^3}\exp\left(-\frac{3}{2}-\frac{1}{k^2\lambda_e^2}\right).$$

In order to have a significant contribution by the damping due to the charging process, the following condition $|\omega_1|>|\Gamma_L|$ must be satisfied. For the example given above, one can find that for $0 \leqslant k\lambda_{\mathrm{De}} < 0.4$ the damping due to the charging process can dominate over the Landau damping.

Finally, consider the case when there is a nonzero electron streaming velocity. The perturbative solution of Eq. (17) can be obtained as before and is given by

$$\omega_0 = \vec{k}\cdot\overrightarrow{V_{e0}} \pm \omega_L \tag{22}$$

and

$$\omega_1 = -i\frac{\beta_{e0}\omega_0}{\omega_0-\vec{k}\cdot\vec{V}_{e0}} + i\frac{\beta_{e0}\omega_{pe}^2}{2\omega_0(\omega_0-\vec{k}\cdot\vec{V}_{e0})} + i\frac{3\beta_{e0}\vec{k}\cdot\overrightarrow{V_{e0}}}{2(\omega_0-\vec{k}\cdot\overrightarrow{V_{e0}})}, \tag{23}$$

where $\omega_L = \sqrt{\omega_{pe}^2 + k^2V_{te}^2}$. In the absence of any streaming velocity Eq. (23) reduces to Eq. (21) and describes the damping of the longitudinal plasma oscillations in a dusty plasma. For $\omega_0 = \vec{k}\cdot\overrightarrow{V_{e0}} - \omega_L$ and $\omega_L \sim \omega_{pe}$, Eq. (23) gives the unstable mode when $(-\omega_L + \sqrt{9\omega_L^2 - 4\omega_{pe}^2})/2 < \vec{k}\cdot\vec{V}_{e0} < \omega_L$. For the other root of Eq. (22), i.e., $\omega_0, = \vec{k}\cdot\vec{V}_{e0} + \omega_L$, the instability occurs when $2\vec{k}\cdot\overrightarrow{V_{e0}} < \omega_L - \sqrt{9\omega_L^2 - 4\omega_{pe}^2}$ or $2\vec{k}\cdot\vec{V}_{e0} > \omega_L + \sqrt{9\omega_L^2 - 4\omega_{pe}^2}$. This instability is driven by the coupling of the attachment frequency and the streaming velocity and may have a similar origin as that discussed in Refs. [10] and [15].

In conclusion, I have studied longitudinal plasma oscillations by using the equations of self-consistent grain charge variations. In the absence of the charging frequency terms in the electron dynamical equations, we have reproduced the results of the previous researchers indicating the instability of the oscillations. In addition, the reason for such instability is explained. It is demonstrated that the instability of longitudinal plasma oscillations disappears when self-consistent grain charge dynamics is taken into account. Furthermore, the present work shows that the use of *ad hoc* (not self-consistent) charge dyanmics can lead to qualitatively incorrect results. Also, the case of nonzero electron streaming velocity in the stationary dust and ion background is studied.

[1] D. A. Mendis and M. Rosenberg, Annu. Rev. Astron. Astrophys. **32**, 419 (1994).
[2] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990).
[3] R. Bingham, U. deAngelis, V. N. Tsytovich, and O. Havnes, Phys. Fluids B **3**, 811 (1991).
[4] P. K. Shukla and V. P. Silin, Phys. Scr. **45**, 508 (1992).
[5] M. Nambu, P. K. Shukla, and S. V. Vladimirov, Phys. Lett. A **180**, 441 (1993).
[6] P. K. Shukla, M. Y. Yu, and R. Bharthuram, J. Geophys. Res. **96**, 343 (1991).
[7] N. D'Angelo, Planet. Space Sci. **42**, 507 (1994).
[8] F. Melandso, T. K. Aslakson, and O. Havens, Planet. Space Sci. **41**, 321 (1993).
[9] R. K. Varma, P. K. Shukla, and V. Krishan, Phys. Rev. E **47**, 3612 (1993).
[10] M. R. Jana, A. Sen, and P. K. Kaw, Phys. Rev. E **48**, 3930 (1993).
[11] V. N. Tsytovich and O. Havnes, Comments Plasma Phys. Controlled Fusion **15**, 267 (1993).
[12] S. V. Vladimirov, Phys. Plasma **1**, 2762 (1994).
[13] J. X. Ma and M. Y. Yu, Phys. Rev. E **50**, R2431 (1994).
[14] F. Li, O. Havnes, and F. Melandso, Planet. Space Sci. **42**, 401 (1994).
[15] J. R. Bhatt and B. P. Pandey, Phys. Rev. E **50**, 3980 (1994).
[16] O. Havnes and G. E. Morfill, Adv. Space Res. **4**, 85 (1984).